Indices, Surds and Logarithms

Indices

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $a^{-n} = \frac{1}{a^n}$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$
- $\frac{ab^{-n}}{c^{-m}d} = \frac{ac^m}{b^n d}$
- $a^0 = 1$
- $a^{\frac{1}{2}} = \sqrt{a}$
- $a^{\frac{1}{3}} = \sqrt[3]{a}$
- $a^{\frac{2}{3}} = \sqrt[3]{a^2}$
- $a^m \times b^m = (ab)^m$

Solving by substitution

Express the indices into its component powers before doing the substitution

Example:
By using a suitable substitution, solve the following equation:

$$3^{2x+3} + 3^{x+3} = 1 + 3^x$$
$$3^3(3^{2x}) + 3(3^x) = 1 + 3^x$$
$$27(3^{2x}) + 2(3^x) - 1 = 0$$

Let $y = 3^x$

$$27y^2 + 2y - 1 = 0$$
$$y = 0.1589 \ or - 0.2330 \ (N.A)$$
$$(Exponentials \ are \ always \ positive)$$
$$3^x = 0.1589$$

Take "lg" both sides

$$x \lg 3 = \lg 0.1589$$
$$x = \frac{\lg 0.1589}{\lg 3} = 0.4771$$

Surds

Common Laws

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$
$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$
$$\sqrt{a} \times \sqrt{a} = a$$
$$a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$$
$$(a + \sqrt{b})(c + \sqrt{d}) = ac + a\sqrt{d} + c\sqrt{b} + \sqrt{bd}$$
$$(1 + \sqrt{a})(1 - \sqrt{a}) = 1^2 - (\sqrt{a})^2 = 1 - a$$

Rationalising the denominator

To remove the surd from the denominator, multiply the entire fraction by the conjugate surd

Example:

$$\frac{3}{4 - 2\sqrt{6}} = \frac{3}{4 - 2\sqrt{6}} \times \frac{4 + 2\sqrt{6}}{4 + 2\sqrt{6}}$$
$$= \frac{12 + 6\sqrt{6}}{4^2 - (2\sqrt{6})^2}$$
$$= \frac{6(2 + \sqrt{6})}{16 - 4(6)}$$
$$= -\frac{3(2 + \sqrt{6})}{4}$$

Simplfying surds

Split the surd into a square root of a perfect square times another surd:

Examples:

$$\sqrt{200} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2}$$
$$\sqrt{768} = \sqrt{256} \times \sqrt{3} = 16\sqrt{3}$$
$$\sqrt{125} = \sqrt{25} \times \sqrt{5} = 5\sqrt{5}$$

Logarithms

Common Laws

$$\log_a b + \log_a c = \log_a(bc)$$
$$\log_a b - \log_a c = \log_a\left(\frac{b}{c}\right)$$
$$\log_a b^c = c \log_a b$$
$$\log_a c = \frac{\log_b a}{\log_b c} \text{ (b can take any value)}$$

Converting from log from to index Form

$$\log_a b = c \quad \leftrightarrow \quad b = a^c$$

Example:

$$\log_2 8 = 3 \quad \leftrightarrow \quad 8 = 2^3$$

Common & Natural logarithms

$$\log_{10} a = \lg a$$
$$\log_e a = \ln a$$

(Use "ln" when there is e ; otherwise always use "lg")

Solving Equations

2 Scenarios to reach:
1. A single log on both sides with the same base

Example:

$$\log_3(2x + 1) = \log_3(x - 3)$$
$$2x + 1 = x - 3$$
$$x = \frac{-3 - 1}{2} = -2$$

2. A single log on one side of the equation

Example:

$$\log_3(2x + 1) = 2$$
$$2x + 1 = 3^2$$
$$x = \frac{9 - 1}{2} = 4$$